Answer Key

ACT2020, MIDTERM #2
ECONOMIC AND FINANCIAL APPLICATIONS
MARCH 16, 2009
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You have 70 minutes to complete this exam. When the invigilator instructs you to stop writing you must do so immediately. If you do not abide by this instruction you will be penalised. All invigilators have full authority to disqualify your paper if, in their judgement, you are found to have violated the code of academic honesty.

Each question is worth 10 points. Provide sufficient reasoning to back up your answer but do not write more than necessary.

This exam consists of 8 questions. Answer each question on a separate page of the exam book. Write your name and student number on each exam book that you use to answer the questions. Good luck!

Question 1 (Text question 5.10)**.** The S&R index spot price is 1100 and the continuously compounded risk-free rate is $r = 0.05$. You observe a 9-month forward price of 1129.257.

(a) [4 points] What dividend yield is implied by the forward price?

(b) [6 points] Suppose you believe the dividend yield over the next 9 months will be only 0.5%. What arbitrage would you undertake?

Question 2. Explain the notion of a credit default swap. Be sure to explain the roles of the counterparties and what payments are made and received by each.

Question 3. You have been offered an opportunity to take a long or short position on the following deal.

- In 6 months time the long pays \$1026.20 for the asset.
- The long receives the asset from the short exactly 2 months after the \$1026.20 payment is made.
- The asset will pay a cash dividend of \$10.50 exactly one month after the payment of \$1026.20 is made.

The continuously compounded interest rate is 4.25% (*i.e.* $r = 0.0425$) and the current market price of the asset is \$1000.00.

(1) [5 points] If profit is your sole motive, is there an attractive opportunity here? Explain which side of the transaction you would take (*i.e.* long or short) and why.

(2) [5 points] Explain how you would hedge out your risk in the transaction you chose in **(1)** and compute what your profit is at the time the asset is delivered to the long.

Question 4. Zero-coupon risk-free bonds are available with maturities of one, two, and three years with corresponding effective annual yield rates:

$$y(1) = 0.060, y(2) = 0.065, y(3) = 0.070$$

where $y(k)$ is the effective annual yield rate for the zero-coupon bond maturing in k years.

You need to buy corn for producing ethanol. You want to purchase 10,000 bushels one year from now, 15,000 bushels two years from now, and 20,000 bushels three years from now. The current forward prices, per bushel, are \$3.89, \$4.11, and \$4.16 for one, two, and three years respectively.

You want to enter into a commodity swap to lock in these prices. Which of the following sequences of payments at times one, two, and three will NOT be acceptable to you and to the corn supplier?

A. 38,900, 61,650, 83,200

B. 39,083, 61,650, 82,039

C. 40,777, 61,166, 81,554

D. 41,892, 62,340, 78,997

E. 60,184, 60,184, 60,184

Question 5 (Text question 5.2)**.** A \$50 stock pays a \$1 dividend every 3 months, with the first dividend coming 3 months from today. The continuously compounded risk-free interest rate is $r = 0.06$.

What is the price of a prepaid forward contract that expires 1 year from today, immediately after the fourth-quarter dividend?

Question 6. Barkley Corporation requires dried chicken treats as an input to production. Barkley Corporation requires 100 bags of dried chicken treats in three months time. The continuously compounded interest rate is $r = 0.03$ and the current price of dried chicken treats is \$2.40 per bag. The CEO of Barkley Corporation, Ms. Barkley, is concerned about a rise in the price of dried chicken treats in three months time.

Ms. Barkley has stated that the price that Barkley Corporation will pay to purchase dried chicken treats in three months time cannot exceed \$3.00 per bag. Ms. Barkley has consulted her CFO, Laddie, and Laddie has come up with the following helpful strategy that he has called "Strategy 1". Strategy 1 involves the idea of collars, something he feels Ms. Barkley will identify with. Barkley Corporation will purchase 100 call options on dried chicken treats expiring in three months with a strike price of \$3.00. Additionally, Barkley Corporation will sell 100 put options on dried chicken treats expiring in three months with a strike price of \$2.0053. The strike price on the put options has been selected so that the market price of the calls is equal to the market price of the puts.

Laddie has also come up with a second strategy that he calls "Strategy 2". Under this strategy, the price that Barkley Corporation will pay to purchase dried chicken treats in three months time may exceed \$3.00 per bag but Laddie wants to offer Ms. Barkley a second option. Strategy 2 involves the use of a sophisticated short and long call position in which Barkley Corporation purchases 200 call options on dried chicken treats expiring in three months with a strike price of \$3.00 and sells 100 call options on dried chicken treats expiring in three months with a strike price of \$2.6784. The nice thing about Strategy 2 is that there is no additonal cost to Barkley Corporation relative to what they would pay if they did not hedge providing the price of dried chicken treats does not increase much over the next three months. Laddie is told that Strategy 2 is what they call a "paylater" strategy.

You are given the following information about the current market prices for options on dried chicken treats.

$$C(2.5) = 0.20427, \quad C(2.6784) = 0.14302, \quad C(3) = 0.07151$$

$$P(3) = 0.64909, \quad P(2.0053) = 0.07151$$

where $P(k)$ is the current market price for a put option on one bag of dried chicken treats expiring in three months with a strike price of k and $C(k)$ is the current market price for a call option on one bag of dried chicken treats expiring in three months with a strike price of k.

(1) [2 points] Draw a chart showing the total cost to Barkley Corporation for the purchase of 100 bags of dried chicken treats in three months time under Strategy 1. Illustrate the chart over a price range of \$0 to \$5 per bag.

(2) [2 points] Draw a chart showing the total cost to Barkley Corporation for the purchase of 100 bags of dried chicken treats in three months time under Strategy 2. Illustrate the chart over a price range of \$0 to \$5 per bag.

(3) [2 points] If the market price for a bag of dried chicken treats turns out to be \$2.75 in three months time, which of the two strategies will have performed better and by how much?

(4) [2 points] If the market price for a bag of dried chicken treats turns out to be \$1.50 in three months time, which of the two strategies will have performed better and by how much?

(5) [2 points] If the market price for a bag of dried chicken treats turns out to be \$3.50 in three months time, which of the two strategies will have performed better and by how much?

Question 7. Explain the notion of an interest rate swap. Be sure to explain the roles of the counterparties and what payments are made and received by each.

Question 8. The current prices of zero-coupon bonds maturing for \$1 at time k are:

- $P(1) = 0.9851$
- $P(2) = 0.9656$
- $P(1) = 0.9418$
- $P(1) = 0.9194$

You are considering a four year interest rate swap with annual payments in arrears.

What is the swap rate on this four year interest rate swap?

Question 1 (Text Question 5.10.)

a) We plug the continuously compounded interest rate, the forward price, the initial index level and the time to expiration in years into the valuation formula and solve for the dividend yield:

$$
\begin{aligned}
& F_{0,T} = S_0 \times e^{(r-\delta)\times T} \\
\Leftrightarrow \quad & \frac{F_{0,T}}{S_0} = e^{(r-\delta)\times T} \\
\Leftrightarrow \quad & \ln\left(\frac{F_{0,T}}{S_0}\right) = (r-\delta)\times T \\
\Leftrightarrow \quad & \delta = r - \frac{1}{T}\ln\left(\frac{F_{0,T}}{S_0}\right) \\
\Rightarrow \quad & \delta = 0.05 - \frac{1}{0.75}\ln\left(\frac{1129.257}{1100}\right) = 0.05 - 0.035 = 0.015
\end{aligned}
$$

Remark: Note that this result is consistent with exercise 5.6., in which we had the same forward prices, time to expiration etc.

b) With a dividend yield of only 0.005, the fair forward price would be:

$$F_{0,T} = S_0 \times e^{(r-\delta)\times T} = 1{,}100 \times e^{(0.05-0.005)\times 0.75} = 1{,}100 \times 1.0343 = 1{,}137.759$$

Therefore, if we think the dividend yield is 0.005, we consider the observed forward price of 1,129.257 to be too cheap. We will therefore buy the forward and create a synthetic short forward, capturing a certain amount of \$8.502. We engage in a reverse cash and carry arbitrage:

Description	Today	In 9 months
Long forward	0	$S_T - \$1{,}129.257$
Sell short tailed position in index	$\$1{,}100 \times .99626 = \$1{,}095.88$	$-S_T$
Lend \$1,095.88	$-\$1{,}095.88$	\$1,137.759
TOTAL	0	\$8.502

Question 2. A credit default swap is an insurance policy against the default of a corporate bond.

There are two parties to the contract. The party that owns the credit default swap is the buyer of the insurance, otherwise knows as the **insured**. The party that sells the credit default swap is the seller of insurance, otherwise known as the **insurer**.

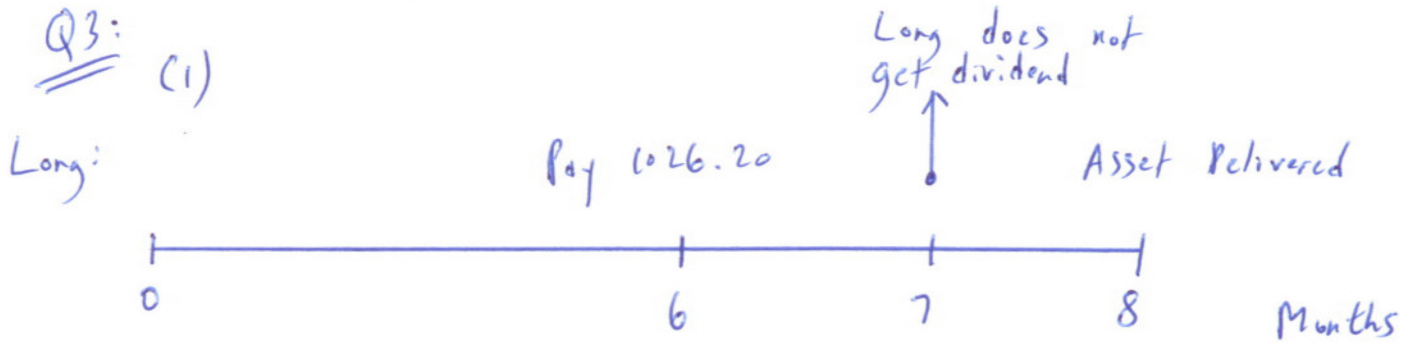
The insured pays a periodic premium for the insurance. The premium might be paid annually, semi-annually, or quarterly. The premium is due so long as the insured bond has not defaulted. If a default event occurs, the insurer takes ownership of the defaulted bond and pays the insured the market value of an otherwise equivalent treasury bond. For example, if default occurs at time t and at the time of default the corporate bond has remaining coupon payments of c scheduled to be paid at times $\tau_k, \tau_{k+1}, \ldots, \tau_N$ and redemption of F scheduled to be paid at time τ_N then the insurer is obliged to pay the insured the amount

$$\text{(CDS)} \qquad c \cdot P(t, \tau_k - t) + c \cdot P(t, \tau_{k+1} - t) + \cdots + (c + F) \cdot P(t, \tau_N - t)$$

where $P(t, s)$ denotes the price at time t of a certain amount of \$1 to be paid at time $t + s$. At the time the insurer pays the preceding amount to the insured the insurer receives ownership of the defaulted bond. The ultimate loss the insurer takes in the event of default depends on the amount that they are able to recover from the defaulted bond. In effect, the insured is obligated to pay the market value of an otherwise equivalent treasury in the event that a default event is triggered and their loss at the time of default will be equal to the value in (CDS) less the accumulated value of the premiums they have received less the present value of the amount they ultimately recover from the defaulted bond they now own. □

Q3:

(1)



The long on this deal is taking a long forward position on the asset with a forward price of

$$1026.20\, e^{.0425\left(\frac{2}{12}\right)} = 1033.49.$$

[The forward price is "prepaid" 2 months before asset delivery.]

The arbitrage-free forward price is:

$$1000\, e^{.0425\left(\frac{8}{12}\right)} - 10.50\, e^{.0425\left(\frac{1}{12}\right)} = 1018.20$$

$\therefore$ there is an attractive opportunity in taking a short position on the deal.

i.e. **Short the deal**

(2) Hedge out risk by borrowing 1000 and buying asset. At delivery you owe $1000\, e^{.0425\left(\frac{8}{12}\right)} = 1028.74$ and you collected a dividend worth $10.50\, e^{.0425\left(\frac{1}{12}\right)} = 10.54$ at delivery. Your net position is:

$$1026.20\, e^{.0425\left(\frac{2}{12}\right)} - \left[1000\, e^{.0425\left(\frac{8}{12}\right)} - 10.50\, e^{.0425\left(\frac{1}{12}\right)}\right]$$

$$= 1033.49 - [1028.74 - 10.54] = \$15.29$$

Summary:

In (1) you shorted the deal. You have therefore agreed to deliver the asset at 8 months in return for a payment of 1026.20 at 6 months. By the time you deliver the asset the dividend has been paid.

To hedge, buy the asset. You need to borrow to do this. At delivery date you owe 1000 plus interest less accumulated value of any dividends you got from the asset you owned prior to delivery.

Net profit at Delivery

= Accumulated Value of Payments Received − Net Cost of Delivering the Asset

$$= 1026.20\, e^{.0425\left(\frac{2}{12}\right)} - \left[1000\, e^{.0425\left(\frac{8}{12}\right)} - 10.50\, e^{.0425\left(\frac{1}{12}\right)}\right]$$

= 15.29.

Answer: Borrow to Buy Asset and Profit = 15.29.

Question 4

Maturity	Eff. Ann. Yield	P(k)	Forward Prices	# Bushels	
1	0.06	0.94340	3.89	10,000.00	$ 36,698.11
2	0.07	0.88166	4.11	15,000.00	$ 54,354.29
3	0.07	0.81630	4.16	20,000.00	$ 67,915.98
					$ 158,968.39

The present value of the forward prices required to obtain the required bushels of corn is: $158, 968.
Therefore, any sequence of payments that has $158,968 as its present value is a fair set of swap payments for the corn required.

		Candidate Payment Sequence				
Maturity	P(k)	A	B	C	D	E
1	0.94340	38,900	39,083	40,777	41,892	60,184
2	0.88166	61,650	61,650	61,166	62,340	60,184
3	0.81630	83,200	82,039	81,554	78,997	60,184
		158968	158193	158969	158968	158967

Answer = B.

Question 5

a) The owner of the stock is entitled to receive dividends. As we will get the stock only in one year, the value of the prepaid forward contract is today's stock price, less the present value of the four dividend payments:

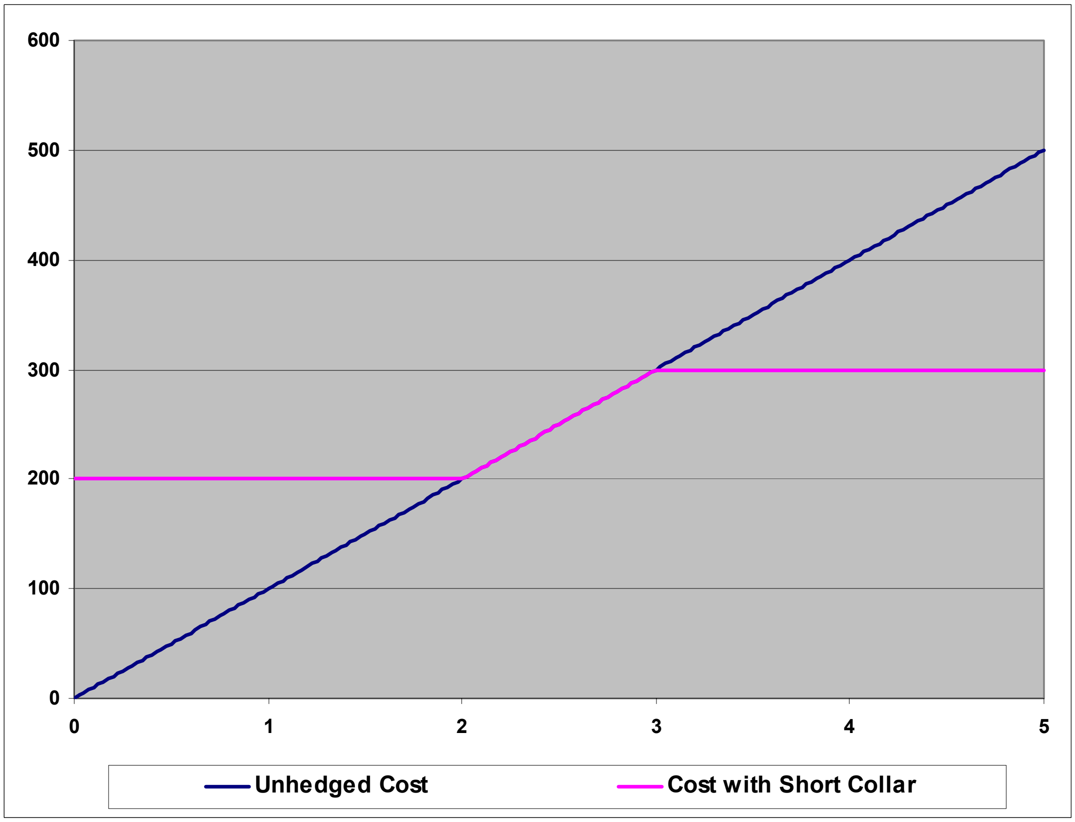
$$F_{0,T}^{P} = \$50 - \sum_{i=1}^{4} \$1 e^{-0.06\times\frac{3}{12}i} = \$50 - \$0.985 - \$0.970 - \$0.956 - \$0.942$$
$$= \$50 - \$3.853 = \$46.147$$

One could also compute the forward price and then adjust for interest
The forward price nad prepaid forward price are related by the following equation:

$$F_{0,T} = F_{0,T}^{P} \times e^{0.06\times 1} = \$46.147 \times e^{0.06\times 1} = \$46.147 \times 1.0618 = \$49.00$$

Question 6

(1) If the price if a bag of dried chicken treats is below \$2.0053 it will cost Barkley Corporation \$2.0053 per bag and thus \$200.53 for the 100 bags. If the price if a bag of dried chicken treats is above \$3.00 it will cost Barkley Corporation \$3.00 per bag and thus \$300.00 for the 100 bags. If the price if a bag of dried chicken treats is x and x is between \$2.0053 and \$3.00 it will cost Barkley Corporation \$x per bag and thus \$100x for the 100 bags. Consequently, the chart for this is shown in the following picture.



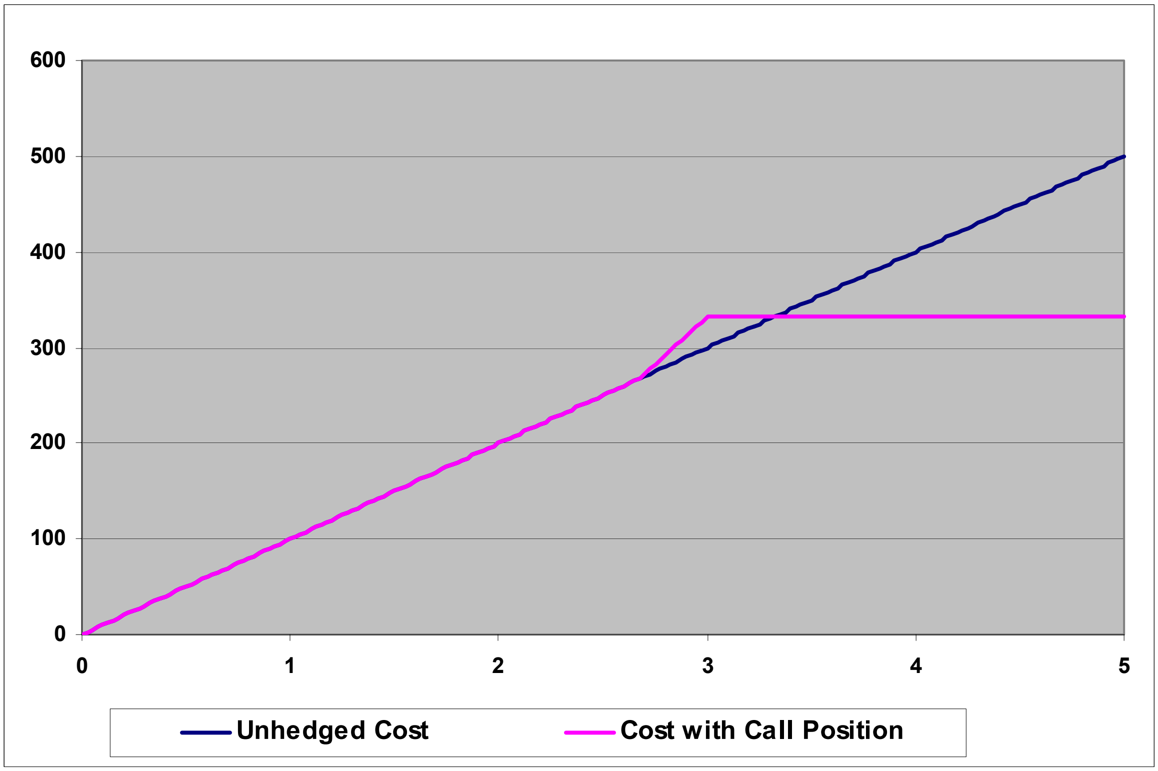
(2) As was the case in (1), the total cost of the option position is 0. Therefore, the cost of purchasing the 100 bags of dried chicken treats does not depend on the cost of options with interest since 200C(3)-100C(2.6784) = 0. Thus, one needs only to graph

$$100S_5 + 100(S_5 - 2.6784)_+ - 200(S_5 - 3)_+ .$$

The chart for the total cost of purchasing 100 bags of dried chicken treats is shown in the following picture. The pink curve is a line of slope 100 (line of slope 1 scaled by 100) until the price of a bag of dried chicken treats hits \$2.6784. From \$2.6784 to \$3.00 the pink curve is a line of slope 200 and above \$3.00 the pink curve is a horizontal line at a height of \$332.16.

We can check the height of the curve beyond \$3.00 as:

$$100S_5 + 100(S_5 - 2.6784) - 200(S_5 - 3) = \$332.16.$$



(3) Strategy 1 will cost \$275 for the 100 bags. Strategy 2 will cost

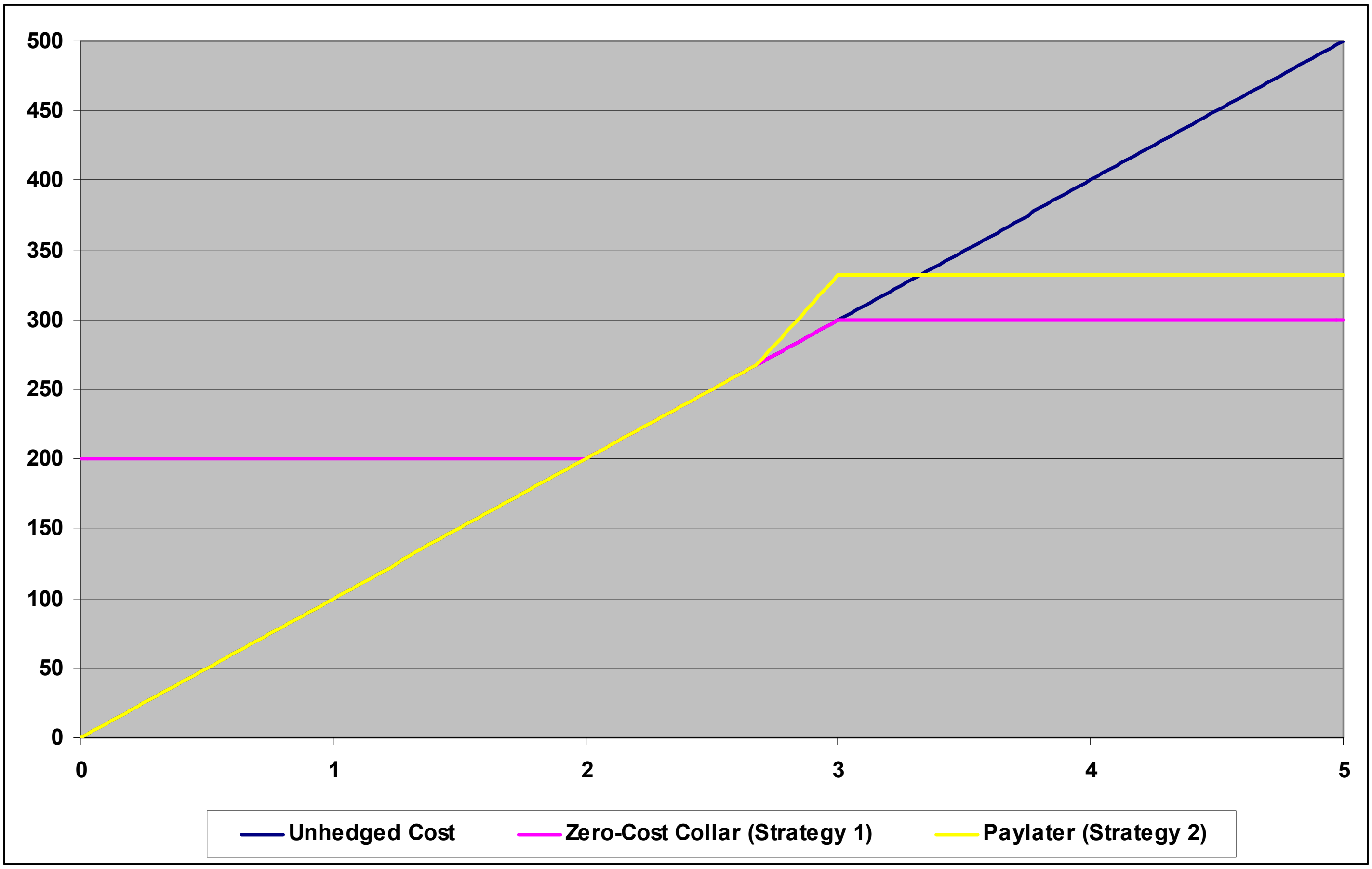
$$100(2.75) + 100(2.75 - 2.6784)_{+} = 282.16 .$$

Therefore, Strategy 1 does better by \$7.16.

(4) Strategy 1 will cost \$200.53 for the 100 bags. Strategy 2 will cost \$150. Therefore, Strategy 2 outperforms by \$50.53.

(5) Strategy 1 costs \$300 and Strategy 2 costs \$332.16. Therefore, Strategy 1 outperforms by \$32.16.

The following picture shows the total cost of purchasing 100 bags of dried chicken treats under each of the two strategies.



Question 7

See test section 8.2.

Question 8

Maturity	Yield	P(Maturity)
1	0.0150	0.9851
2	0.0175	0.9656
3	0.0200	0.9418
4	0.0210	0.9194

R	0.02114	=(1-D9)/SUM(D6:D9)

Answer = 2.114%